Num. Methods in CAE – WS 18/19 – Short solutions

Exercise 1 (7 points):

(a) $\boldsymbol{f}(x,y) \;=\; \begin{pmatrix} x^4+(y-1)^4-1 \\ (x-2)^2+y^2-3 \end{pmatrix}$

(b) $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 17/64 \\ 1/1024 \end{pmatrix}$

Exercise 2 (11 points):

(a) $a > 2$ or $a < -2$

(b) $\|\boldsymbol{x}^* - \boldsymbol{x}^{(k)}\|_\infty < 2 \cdot 10^{-5}$

(c) 7 additional steps.

Exercise 3 (15 points):

(a) $c_0 \approx \frac{3}{4}, \quad c_{\pm 1} = \frac{1}{\pi} \mp \frac{j}{2} \approx -\frac{1}{3} \mp \frac{j}{2}, \quad c_{\pm 2} = \pm\frac{j}{4}$

(b) $a_0 = \frac{\pi}{2}, \quad a_k = \frac{(-1)^k - 1}{\pi k^2}, \quad b_k = \frac{(-1)^{k+1}}{k}, \quad k \in \mathbb{N}$

(c) $S_{2,g}(x) = \frac{3\pi}{4} + \frac{2}{\pi}\cos(x) - \sin(x) + \frac{1}{2}\sin(2x)$

$S_{2,h}(t) = \frac{3}{4} + \frac{2}{\pi^2}\cos(\pi t) - \frac{1}{\pi}\sin(\pi t) + \frac{1}{2\pi}\sin(2\pi t)$

Exercise 4 (15 points):

(a) $p_2(x) = 1 + \frac{x}{2} + \frac{3x^2}{8}, \qquad R_2(x) = \frac{5}{16}\,\frac{x^3}{\sqrt{1-\xi}^{\,7}}, \quad \xi$ between 0 and x.

(b) $\tilde{I} = \frac{297}{512}\,\pi$

(d) $I - \tilde{I} \leqslant \frac{25\sqrt{2}\pi}{512}$

Exercise 5 (12 points):

(a) Explicit 3-stage Runge Kutta method, hence $p = 3$.

(b) $w_1 = -1 + \frac{h^2}{2} + \frac{h^3}{6}$

(c) $y(t) = -1 + \frac{(t-1)^2}{2!} + \frac{(t-1)^3}{3!} + \dots$

(d) $\tilde{w}_1 = -1 + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!}$ (no higher order terms!)