Hochschule Esslingen – University of Applied Sciences

Master Course Design and Development of Automotive and Mech. Engineering		
Module:	**Numerical Methods in CAE**	Page 1 of 3
Semester:	Winter 2017/18	Time: 90 mins
Remarks:	Notes & documents from the lecture and 10 pages of personal notes allowed No calculator or other electronic devices allowed	

Name ______________________ First name ______________________ Matr.-Number ______________

Note: 57 points can be achieved in total.

Exercise 1 (9 points):

We consider the linear equation system $\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{b}$ with

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & a & c \\ 0 & c & 12 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \qquad \boldsymbol{x}^{(0)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix};$$

herein, a and c are real parameters.

(a) For which values of a, c is $\mathbf{A}$ strictly row diagonal dominant?

(b) For which values of a, c is $\mathbf{A}$ positive definite?

(c) Suppose $a = 2,\ c = -2$. Would you rather use the method of the steepest descent (SD) or Gauß-Seidel's method to solve the LES?

(d) **Let $\boldsymbol{a} = 4$ and $\boldsymbol{c} = 1$.** Perform one step of the SD method in order to compute the first approximation $\boldsymbol{x}^{(1)}$ to the exact solution of the LES.

Exercise 2 (15 points):

We consider the function

$$f(t) := \frac{1}{1 - e^{-2\pi}} \cdot e^{-t}, \quad t \in [0, 2\pi) \qquad \text{and extended } 2\pi\text{−periodically.}$$

(a) Compute the complex Fourier coefficients $c_k,\ k \in \mathbb{Z}$ of the Fourier series of f.

(b) Sketch c_0 and $c_{\pm 1}$ in the complex plane.

(c) Use c_k to compute the real Fourier coefficients a_k, b_k. Write down the real Fourier series of f up to terms of order $k = 2$ inclusively.

(d) What is the mean value of f?

Exercise 3 (10 points):

We want to solve the initial value problem

$$\begin{cases} y'(t) &= y(t)^2 - t \\ y(1) &= -1 \end{cases}$$

using the $3^{rd}-$order Runge Kutta method which is defined by the Butcher table depicted to the right and step size h.

$$\begin{array}{c|ccc} \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \end{array}$$

(a) Compute the approximation w_1 for the exact solution $y(t_1)$ after the first step. Also write down t_1 as a function of h.

(b) Write down as many terms of the Taylor series with center point 1 of the exact solution $y(t)$ as you know from the results of exercise part **(a)**.

Exercise 4 (8 points):

(a) Use the Taylor series for the exponential function and the cosine function in order to compute the Taylor polynomial *of degree 6* for

$$f(x) := e^{-x^2} \cos(x)$$

with center point $x_0 = 0$.

(b) Use the Taylor polynomial *of degree 4* of exercise part **(a)** in order to compute an approximation for

$$\int_0^1 e^{-x^2} \cos(x)\, dx\,.$$

(c) Use the previous exercise parts in order to obtain an interval I such that

$$\int_0^1 e^{-x^2} \cos(x)\, dx \in I.$$

Hints:

- You may assume that the series under investigation meets the requirements for Leibniz' criterion.
- Use $331/720 < 7/15$.

Exercise 5 (15 points):

In order to determine the intersection point of the curves $y = x$ and $y = \cos(x)$ we apply Newton's method to the function

$$f(x) \;:=\; x - \cos(x)\,.$$

(a) Show that there exists a unique number $x^* \in [0, \pi/2]$ such that $f(x^*) = 0$.

(b) Show that x^* has multiplicity 1. What does that mean for Newton's method ?

(c) Beginning with $x_0 := \pi/2$, perform one step of Newton's method.

(d) Compute the iteration function F.

- Show *computationally* that F is contracting on the interval $[0, 1]$.
- Use the pictures below to show *geometrically* that F is also contracting on the interval $I \;:=\; [\,0.6\,,\,1\,]$.
- Which information do you need additionally (and can be obtained from the pictures) in order to conclude that Newton's method converges on I ?

(e) Use the pictures below to give an estimate for x^* without computation.

(f) Suppose you have computed an approximation $x_n \in I$ with $|x_n - x^*| < 0.1$. Use the above exercise parts and the pictures below in order to make a statement on the accuracy of the successive approximation x_{n+1}.