Hochschule Esslingen – University of Applied Sciences

Master Course Design and Development of Automotive and Mech. Engineering		
Module:	**Numerical Methods in CAE**	Page 1 of 2
Semester:	Winter 2016/17	Time: 90 mins
Remarks:	Notes & documents from the lecture and 10 pages of personal notes allowed No calculator or other electronic devices allowed	

Name ______________ First name ______________ Matr.-Number ______________

Note: 57 points can be achieved in total.

Exercise 1 (9 points):

Consider the function

$$\boldsymbol{f}(x,y) \;:=\; \begin{pmatrix} (x^2-1)(y^2-1) \\ (x^3-3x)y \end{pmatrix} .$$

(a) Beginning with $(x_0, y_0) = (2, 1)$, perform one step of Newton's method in order to obtain an approximation for one of the zeros of f.

(b) It can be shown that for $(x_0, y_0) = (0, a)\,,\ a \neq 0$ Newtons method converges to a zero (x^*, y^*) of $\boldsymbol{f}$. Compute (x_1, y_1). What can you conclude for the exact value x^* ? (Give a *short* argument.)

Exercise 2 (20 points):

We consider the linear equation system $\mathbf{A}\cdot\boldsymbol{x} \;=\; \boldsymbol{b}$ with

$$\mathbf{A} \;=\; \begin{pmatrix} 4 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \qquad \boldsymbol{b} \;=\; \begin{pmatrix} 10 \\ 9 \\ 10 \end{pmatrix}.$$

(a) Show that the solution of this LES is unique. (It is *not* required to compute the exact solution !)

(b) Show that Gauß-Seidel's method for solving the LES iteratively converges.

(c) Beginning with $\boldsymbol{x}^{(0)} \;=\; (1, 2, 4)$ perform one step of Gauß-Seidel's method in order to obtain an approximation $\boldsymbol{x}^{(1)}$ to the exact solution of the LES.

(d) We want to obtain an approximation $\boldsymbol{x}^{(n)}$ which differs by no more than 10^{-4} from the exact solution of the LES in the maximum norm. Determine the number n of iteration steps which are required at most to obtain this accuracy (*à priori* estimate).

Use $\lg(2) \approx 0.3$

(e) Would it be better or worse to use the method of the steepest descent instead of Gauß-Seidel's method ? Give a *short* argument.

Exercise 3 (14 points):

We consider a $2\pi-$periodic function $f_p : \mathbb{R} \longrightarrow \mathbb{R}$ which depends on a real parameter $p \in \mathbb{R}$. The complex Fourier coefficients of f_p are given by

$$c_0 = \frac{p\pi^2}{6}, \qquad c_k = \frac{p}{k^2}(-1)^k + jp\Big(\frac{\pi}{2}\frac{(-1)^k}{k} + \frac{1-(-1)^k}{\pi k^3}\Big), \quad k \in \mathbb{Z}^*.$$

(a) Sketch the pointers of $c_0, c_{\pm1}$ and $c_{\pm2}$ in the complex plane (use $\pi \approx 3$ and $p = 4$).

(b) What can you deduce from the c_k regarding symmetry and continuity of f_p ?

(c) What do you know about the relationship between c_k and c_{-k} for $k \in \mathbb{N}$, given the fact that f_p is a real function ?

(d) Compute the real Fourier coefficients a_k, b_k.

(e) Determine p such that the mean value of f_p is $\overline{f}_p = 1$.

(f) The function g is related to f_p by the equation

$$g(t) := \frac{1}{p} \cdot f_p(4t).$$

Write down the real Fourier series of g up to terms of order $k = 2$ inclusively.

Exercise 4 (14 points):

The initial value problem

$$y' = \frac{2(1+3t)}{y^2}, \qquad y(0) = 1$$

has the solution

$$y(t) = \sqrt[3]{1+3t}^{\,2}.$$

(a) Compute the Taylor polynomial $p_3(t)$ of degree 3 with center point $t_0 = 0$ for y.

(b) Perform one step of the midpoint rule with step size h in order to determine an approximation w_1 for $y(h)$.

Result: $$w_1 = 1 + \frac{h(2+3h)}{(1+h)^2} \qquad (*)$$

(c) The approximation w_1 depends on h. Use $(*)$ in order to determine the Taylor polynomial $\tilde{p}_3(h)$ of degree 3 with center point $h = 0$ for $w_1(h)$.

Hint: Cauchy product, $\frac{1}{1+z} = 1 - z + z^2 - z^3 \pm \ldots$

(d) Explain shortly why the first few terms of the Taylor polynomials of exercise parts **(a)** and **(c)** agree.

Hint: According to the theory: How many terms *must* agree ?