# Num. Methods in CAE – WS 15/16 – Short solutions

**Exercise 1 (11 points):**

**(a)** $a_{11} > 0, \quad a_{22} > 0, \quad a_{32} = -1, \quad a_{33} > \frac{a_{11} + a_{22}}{a_{11}a_{22}}\,; \quad b \in \mathbb{R}$

**(b)** $\boldsymbol{x}^{(1)} = \big(\, 1.0\,,\ 1.5\,,\ 0.5 \,\big)$

**Exercise 2 (6 points):**

**(a)** $t_1 = h\,,\ \ w_1 = 2 - 2h - h^2 + h^3$ **(b)** $a_0 = 2,\ a_1 = -2,\ a_2 = -1$ **(c)** 5 terms

**Exercise 3 (11 points):**

**(a)** $c_0 = \frac{\pi^2}{6}, \qquad c_k = \frac{(-1)^k}{k^2} + j\Big(\frac{\pi}{2}\,\frac{(-1)^k}{k} + \frac{1-(-1)^k}{\pi k^3}\Big)$

**(b)** $a_0 = \frac{\pi^2}{3}, \qquad a_k = 2\,\frac{(-1)^k}{k^2}, \qquad b_k = -\pi\,\frac{(-1)^k}{k} - 2\,\frac{1-(-1)^k}{\pi k^3}\,, \qquad k \in \mathbb{N}$

**(c)** $S_2(x) = \frac{\pi^2}{6} - 2\cos(x) + \Big(\pi - \frac{4}{\pi}\Big)\sin(x) + \frac{1}{2}\cos(2x) - \frac{\pi}{2}\sin(2x)$

**Exercise 4 (15 points):**

**(a)** $p_2(x) = 2 + \frac{1}{4}(x-5) - \frac{1}{64}(x-5)^2$

**(b)** $\sqrt{6} \approx 39/16$

**(c)** $\sqrt{6} \in \Big[\, \frac{39}{16}\,,\ \frac{39}{16} + \frac{1}{64} \,\Big]$

**Exercise 5 (10 points):**

**(a)** $f$ is convex for $x > 0$, and for $x_0 = a$ we have $x_0 > \sqrt[7]{a}\,$.
Depicting some tangents shows that $x_0 > x_1 > x_2 > \ldots > \sqrt[7]{a}\,$.
Since $\{x_n\}$ is bounded and strictly monotonically decreasing, it must converge.

**(b)** $x^* = \sqrt[7]{a}$ has multiplicity 1.

**(c)** $F(x) = \frac{6x^7 + a}{7x^6}, \qquad F'(x) = \frac{6}{7}\Big(1 - \frac{a}{x^7}\Big), \qquad F''(x) = \frac{6a}{x^8}$

**(d)** $|x_{n+1} - x^*| \leqslant 3\,|x_n - x^*|^2$;
$|x_{n+1} - x^*| < 3 \cdot 10^{-6}, \qquad |x_{n+2} - x^*| < 2.7 \cdot 10^{-11}$