Num. Methods in CAE – WS 14/15 – Short solutions

Exercise 1 (9 points):

(a) $det(\mathbf{A}) \neq 0$

(b) Yes, since $\mathbf{A}$ is strictly diagonal dominant.

(c) Yes, since $\mathbf{A}$ is symmetric and positive definite (Sylvester's criterion).

(d) $\boldsymbol{x}^{(1)} = (0, -1, 1)$

Exercise 2 (12 points):

$$c_k = \frac{1}{\pi(1+k^2)} \Big(1 - e^{-\pi}(-1)^k\Big) \quad (k \in \mathbb{Z})$$

$$a_k = \frac{2}{\pi(1+k^2)} \Big(1 - e^{-\pi}(-1)^k\Big) \quad (k \in \mathbb{N}_0)\,; \qquad b_k = 0 \quad (k \in \mathbb{N})$$

$$T_f(t) = \frac{1-e^{-\pi}}{\pi} + \frac{1+e^{-\pi}}{\pi}\cos(t) + \frac{2-2e^{-\pi}}{5\pi}\cos(2t) + \ldots\,, \qquad \overline{f} = \frac{1-e^{-\pi}}{\pi}$$

Exercise 3 (16 points):

(a) f is continuous, strictly monotonically increasing and changes its sign on $[0,1]$.

(b) $x_1 = 1/2$

(c) $F(x) = \frac{x+1}{e^x+1}\,; \qquad F'(x) = \frac{1-xe^x}{(e^x+1)^2}\,.$

For $x \in [0,1]$ we have $0 \leqslant x < e^x$ and therefore $F(x) \in (0,1)$.

Since F' is strictly monotonically decreasing, we obtain $L = 1/4 < 1$ from investigating $|F'(0)|$ and $|F'(1)|$.

(d) $f'(x) \neq 0$ for all x, hence $f'(x^*) \neq 0$.

(e) $|x_0 - x^*| < 1$ and the formula for quadratic convergence yield $|x_3 - x^*| < 1/2^{14}$.

Exercise 4 (16 points):

(a) $w_1 = \frac{h}{6}\Big(1 + \frac{4}{1+h} + \frac{1+h}{1+3h}\Big)\,; \qquad t_1 = 1+h$

(b) $p_3(h) = h - h^2 + \frac{5h^3}{3}$

(c) $y(t) = (t-1) - (t-1)^2 + \frac{5(t-1)^3}{3} + O\big((t-1)^4\big)$