# Num. Methods in CAE – WS 13/14 – Short solutions

**Exercise 1 (9 points):**

**(a)** Yes, since **A** is symmetric and positive definite (Sylvester's criterion or investigation of the sign of the eigenvalues).

**(b)** $\boldsymbol{x}^{(1)} = \left( -\frac{7}{9}\,,\, \frac{2}{9}\,,\, 1 \right)$

**Exercise 2 (14 points):**

**(a)** $Y^{(2)}(1+h) = h + \dfrac{h^2}{2}\,, \qquad Y^{(3)}(1+h) = h + \dfrac{h^2}{2} + \dfrac{h^3}{6}\,; \qquad t_1 = 1+h$

**(b)** $y(t) = (t-1) + \dfrac{(t-1)^2}{2} + \dfrac{(t-1)^3}{6}$

**(c)** $h \in \left[\, 2.5 \cdot 10^{-3}\,,\, 7.5 \cdot 10^{-3} \,\right]$

**Exercise 3 (5 points):**

$(x_1, y_1) = (1.1, 0.8)$

**Exercise 4 (9 points):**

**(a)** $e^{-x^2/2} = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k \cdot k!} \cdot x^{2k}$

**(b)** $I = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k \cdot k! \cdot (2k+1)}$

**(c)** 3 terms (Leibniz' criterion), $I \approx \dfrac{103}{120}$

**Exercise 5 (15 points):**

**(a)** $c_0 = -\frac{\pi}{4}\,, \qquad c_k = \begin{cases} -\dfrac{j}{2k} + \dfrac{1}{\pi k^2}\,, & k = \pm 1, \pm 3, \ldots \\ \dfrac{j}{2k}\,, & k = \pm 2, \pm 4, \ldots \end{cases}$

**(b)** $a_0 = -\frac{\pi}{2}\,, \qquad a_k = \begin{cases} \dfrac{2}{\pi k^2}\,, & k \text{ odd} \\ 0\,, & k \text{ even} \end{cases} \qquad b_k = \begin{cases} \dfrac{1}{k}\,, & k \text{ odd} \\ -\dfrac{1}{k}\,, & k \text{ even} \end{cases}$

$T_f(t) = -\frac{\pi}{4} + \frac{2}{\pi} \cos(t) + \sin(t) - \frac{1}{2} \sin(2t) \pm \ldots$

**(c)** $\overline{f} = -\frac{\pi}{4}$