# Hochschule Esslingen – University of Applied Sciences

<table>
<tr><td colspan="3">Master Course Design and Development of Automotive and Mech. Engineering</td></tr>
<tr><td>Module:</td><td>Numerical Methods in CAE</td><td>Page 1 of 2</td></tr>
<tr><td>Semester:</td><td>Winter 2013/14</td><td>Time: 90 mins</td></tr>
<tr><td>Remarks:</td><td colspan="2">Notes & documents from the lecture and 10 pages of personal notes allowed<br>No calculator or other electronic devices allowed</td></tr>
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Name ______________ First name ______________ Matr.-Number ______________

## Note: 52 points can be achieved in total.

**Exercise 1 (9 points):**

We consider the linear equation system $\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{b}$ with

$$\mathbf{A} = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}.$$

**(a)** Can we guarantee that the method of the steepest descend (SD) converges?

**(b)** Beginning with $\boldsymbol{x}^{(0)} = (-1, 0, 1)$ perform one step of the SD method in order to obtain an approximation $\boldsymbol{x}^{(1)}$ to the exact solution of the LES.

**Exercise 2 (14 points):**

We want to solve the initial value problem

$$\begin{cases} y'(t) &= 1 + y(t) \\ y(1) &= 0 \end{cases}$$

using the embedded midpoint rule which is defined by the Butcher table depicted to the right and step size $h$.

$$\begin{array}{c|ccc} \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & 0 & 1 & \\ & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \end{array}$$

Recall that the embedded midpoint rule consists of a $2^{nd}$−order and a $3^{rd}$−order Runge-Kutta method.

**(a)** Compute the values $Y^{(2)}(t_1)$ and $Y^{(3)}(t_1)$ as functions of $h$ which the two Runge-Kutta methods yield for the approximation of the exact solution $y(t_1)$ after the first step. Also write down $t_1$ as a function of $h$.

**(b)** Write down as many terms of the Taylor series with center point 1 of the exact solution $y(t)$ as you know from the results of exercise part **(a)**.

**(c)** Suppose we are given a tolerance $\varepsilon = 10^{-5}$. How do we have to choose $h$ such that it is accepted for the first step? (Use $\sqrt{6} \approx 2.5$, $\sqrt{10} \approx 3$.)

**Exercise 3 (5 points):**

Consider the function

$$\boldsymbol{f}(x,y) \;:=\; \begin{pmatrix} x^2 - y^3 \\ x^2 + y^2 - 1 \end{pmatrix} .$$

Starting with $(x_0, y_0) = (2, 1)$, perform one step of Newton's method in order to obtain an approximation $(x_1, y_1)$ for one of the zeros of $\boldsymbol{f}$.

**Exercise 4 (9 points):**

**(a)** Compute the Taylor series at $x_0 = 0$ for

$$f(x) \;=\; e^{-x^2/2} .$$

You may use the Taylor series $\qquad e^z \;=\; \sum_{k=0}^{\infty} \frac{z^k}{k!} .$

**(b)** Use the result of exercise part **(a)** to determine a series for

$$I \;:=\; \int_0^1 f(x)\,dx .$$

**(c)** How many terms of the series for $I$ do we have to sum up in order to get an approximation which differs by no more than $10^{-2}$ from the exact value?

Compute the respective approximation.

**Exercise 5 (15 points):**

We consider the function

$$f(t) \;:=\; \begin{cases} t\,, & t \in [-\pi, 0\,)\,, \\ 0\,, & t \in [\,0, \pi\,) \end{cases} \qquad \text{and extended } 2\pi\text{−periodically.}$$

**(a)** Compute the complex Fourier coefficients $c_k\,,\ k \in \mathbb{Z}$ of the Fourier series of $f$.

**(b)** Use $c_k$ to compute the real Fourier coefficients $a_k, b_k$. Write down the real Fourier series of $f$ up to terms of order $k = 2$ inclusively.

**(c)** What is the mean value of $f$?