Surface Interrogation in Computer Aided Geometric Design

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Abstract

Geometric modelling of vehicle bodies like cars, airplanes, ships etc. uses the geometric concepts of freeform spline curves and surfaces. The quality of these models can be analysed with several surface interrogation tools like curvature plots, isophotes, highlight and reflection lines. In this paper several approaches to reflection lines are reviewed. The main aim of the paper is to reveal the applications of a technique developed by the author which relates reflection lines to the contours of a certain nonlinear function.

1 Introduction

The standard geometric models for car bodies are designed with the aid of freeform NURBS (non uniform rational B- spline) curves and surfaces. The theory of NURBS models is extensively described in the Computer Aided Geometric Design (CAGD) literature for instance in [4] and [13]. Industrial implementations of these concept in industrial CAD/CAM software systems were initiated by Mercedes - Benz, Toyota, Dassault Systems and became a worldwide standard. The quality of the geometric models is a crucial issue for industrial applications. The main focus is on aesthetical and aerodynamical features. These can be analysed and quantified with several interrogation tools designed for the analysis of the smoothness of the constitutive splines. Reflection lines belong to the class of first order interrogation tools. In the automobile industry they are defined as the reflection on the car body of a set of parallel linear lights placed over or near the vehicle as can be seen in Figure 1.
Reflection lines should have some aesthetical features like smoothness and symmetry. If they fail, the surface geometry should be slightly modified in order to reveal improved reflections. This technique of surface 'fairing' using reflections was developed in several papers, particularly in [8], [7], [10].

If the surface model is $G^k$ continuous, the reflection lines will be $G^{k-1}$ (see [4]). Surface quality of order $G^2$ or at least $G^1$ is expected producing $G^1$ or $G^0$ reflection lines. Figure 1 shows engineers analysing reflection lines on the car body of an automobile.

![Figure 1: Reflection lines. Image courtesy of Gerald Farin [5].](image)

The analysis of reflection lines is first performed on virtual car models as can be seen in Figure 12.

A series of other surface interrogation tools like isophotes (see [14]), a different type of reflection lines introduced in Kaufmann and Klass (see [8]), highlight lines (see [1]), reflection ovals (see [3]), reflection curves of arbitrary shape (see [16]), circular highlight and reflection lines ([11]) etc. are available. The paper [6] contains a survey of interrogation tools.

This paper is structured as follows. In Section 2 we briefly introduce the formalism used in geometric modelling using the concepts of freeform spline curves and surfaces. In Section 3 we focus on several techniques for the computation and rendering of reflection lines. The main aim of this paper is to reveal the applications of a technique developed by the author which relates reflection lines to the contours of a certain nonlinear function. This issue is addressed in Section 4.
2 Freeform Curves and Surfaces

The formalism used in geometric modelling is based on freeform spline curves and surfaces. The state of the art and the industrial standard is provided by the NURBS concept (NonUniform Rational B-Splines). This concept was developed in several steps summarized in the next subsections (see also [4] and [13]).

2.1 Bézier Curves

Bézier curves of degree \( n \) are defined as

\[
C(t) = \sum_{i=0}^{n} B^n_i(t) \cdot B_i, \quad t \in [0, 1],
\]

where

\[
B^n_i(t) = \binom{n}{i} (1-t)^{n-i} t^i, \quad i = 0, 1, 2, ..., n
\]

are the Bernstein Polynomials and \( B_i \) the Bézier control points.

![Figure 2: A cubic Bézier curve and its Bézier control polygon.](image)

A cubic Bézier curve is represented in Figure 2.

2.2 Bézier Spline Curves

Bézier spline curves are obtained by linking a number of \( l \) Bézier curves at junction points corresponding to the parameter values \( t_k \):

\[
C(t) = \sum_{i=0}^{n} B^n_i \left( \frac{t - t_{k-1}}{t_k - t_{k-1}} \right) \cdot B^k_i, \quad t_{k-1} \leq t < t_k, \quad k = 1, ..., l
\]
A cubic Bézier spline curve is represented in Figure 3.

2.3 B-Spline Functions

We define B-Spline functions as piecewise polynomial $C^{n-1}$-continuous functions of degree $n$. The parameters (breakpoints) of the junction points are:

$$t_0 < t_1 < \ldots < t_l$$

The knots are defined as:

$$s_{k+n} = t_k, \quad k = 1, 2, \ldots, l - 1$$

$$s_0 = s_1 = \ldots = s_n = t_0; \quad s_l+n = s_{l+n+1} = \ldots = s_{l+2n} = t_l$$

The B-Spline basis functions $N^n_i(s)$, $i = 0, 1, \ldots, l + n - 1$ over the above knot sequence are defined iteratively:

$$N^n_0(s) = \begin{cases} 1 & : \quad s_i \leq s < s_{i+1} \\ 0 & : \quad else \end{cases}$$

$$N^n_i(s) = \frac{s - s_i}{s_{i+n} - s_i} N^{n-1}_i(s) + \frac{s_{i+n+1} - s}{s_{i+n+1} - s_{i+1}} N^{n-1}_{i+1}(s)$$

More general continuity degrees at the breakpoints can be defined by introducing multiple knots (see [4] and [13]).
2.4 B-Spline Curves

B-Spline curves of degree $n$ with $l$ pieces are defined as:

$$C(s) = \sum_{i=0}^{l+n-1} N_i^n(s) \cdot D_i, \quad s \in [s_n, s_{l+n}]$$

where $N_i^n$ are the B-Spline functions and $D_i$ the de Boor control points.

![B-Spline Curve](image)

Figure 4: A cubic B-spline curve and its Bézier and de Boor control points.

2.5 Bézier and B-Spline Surfaces

A tensor product Bézier surface patch is defined as:

$$S(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_i^n(u)B_j^m(v) \cdot B_{ij}, \quad (u, v) \in [0, 1] \times [0, 1]$$

A tensor product B-Spline surface is defined as:

$$S(u, v) = \sum_{i=0}^{k+n-1} \sum_{j=0}^{l+m-1} N_i^n(u)N_j^m(v) \cdot D_{ij}, \quad (u, v) \in [u_n, u_{k+n}] \times [v_m, v_{l+m}]$$

The surface is $C^{n-1}$ respectively $C^{m-1}$—continuous at the junctions between neighbouring surface patches.
2.6 Non Uniform Rational B-Splines (NURBS)

By introducing rational coordinate functions and weights $w_i$, a more general type of freeform geometric model, namely the NURBS model is introduced. NURBS is also the industrial standard in CAD/CAM applications. A NURBS curve is defined as:

$$C(s) = \frac{\sum_{i=0}^{l+n-1} w_i N^n_i(s) \cdot D^i}{\sum_{i=0}^{l+n-1} w_i N^n_i(s)}, \quad s \in [s_n, s_{l+n}]$$

A NURBS surface is defined as:

$$S(u, v) = \frac{\sum_{i=0}^{k+n-1} \sum_{j=0}^{l+m-1} N^n_i(u) N^m_j(v) \cdot w_{ij} D^{ij}}{\sum_{i=0}^{k+n-1} \sum_{j=0}^{l+m-1} N^n_i(u) N^m_j(v) w_{ij}}, \quad (u, v) \in [u_n, u_{k+n}] \times [v_m, v_{l+m}]$$

The surface is $C^{n-1}$ respectively $C^{m-1}$—continuous at the junctions between neighbouring surface patches. NURBS geometry allows the exact representation of conics and particularly circles. This is particularly useful for modelling rotation and blending surfaces which are extensively used in CAD/CAM applications.
3 Reflection Lines

3.1 The Reflection Scene

Figure 6 shows the path of the light from the light point to the eye after reflection on the surface:

Figure 6: Point reflections.

Figure 7 shows a reflection line pattern on the surface:

Figure 7: Reflection lines.
Figure 8 introduces some notations and shows a viewpoint point $A$, a light point $P$ on a light line, the corresponding reflection point $R$ and the normal to the surface $R\hat{N}$:

![Figure 8: The Geometry of Reflection.](image)

Using the notations in Figure 8 the following ‘reflection equation’ can be derived:

$$\frac{\vec{p}}{|\vec{p}|} + \frac{\vec{a}}{|\vec{a}|} = 2 \frac{\vec{a} \cdot \vec{n}}{||\vec{a}||\vec{n}|}, \frac{\vec{n}}{|\vec{n}|}$$

(1)

where, $\vec{a} = \vec{RA}$, $\vec{p} = \vec{RP}, \vec{n} = \hat{R}\vec{N}$. Since we are dealing with parametric surfaces $\vec{r} = \vec{r}(u, v)$ as detailed in the previous section, the three vectors involved in the reflection scene can be represented as $\vec{n} = \vec{n}(u, v), \vec{p} = \vec{p}(u, v)$ and $\vec{a} = \vec{a}(u, v)$.

Therefor this equation is a nonlinear equation:

$$h(u, v) := \frac{\vec{p}}{|\vec{p}|} + \frac{\vec{a}}{|\vec{a}|} - 2 \frac{\vec{a} \cdot \vec{n}}{||\vec{a}||\vec{n}|}, \frac{\vec{n}}{|\vec{n}|} = 0.$$  

(2)

The reflection equation can be solved in different ways which will be reviewed in the following subsection.
3.2 Solving techniques for the reflection equation

Nonlinear Minimization

To solve this equation in a numerically stable manner, we transform it into a nonlinear minimization problem. Define \( g(u, v) := |h(u, v)|^2 \). Then equation (2) is equivalent to the minimization of the nonnegative scalar function \( g(u, v) \). Consider a sampling of lightpoints \( P \) on the lightlines. Start the nonlinear minimization with an initial light point \( P_0 \) and find a corresponding reflection point \( R_0 \). When we consider the 'next neighbour' lightpoint \( P_1 \), we can use \( R_0 \) as initial value in the minimization which provides the reflection \( R_1 \). By repeating this scheme, a sampling of points of the reflection lines is obtained.

Raytracing

A ray \( \vec{RP} \) starts in \( R \) and has a direction \( \vec{s} \) that can be derived from the reflection equation (1):

\[
\vec{s} = \frac{\vec{p}}{|\vec{p}|} = 2 \frac{\vec{a} \cdot \vec{n}}{|\vec{a}| |\vec{n}|} \cdot \vec{n} - \frac{\vec{a}}{|\vec{a}|} \tag{3}
\]

Consider also point \( B \) on a light line with direction \( \vec{c} \) and denote \( \vec{b} = \vec{RB} \). The 3D distance between the ray and the light line is

\[
d = \frac{|(\vec{s} \times \vec{c}) \cdot \vec{b}|}{|\vec{s} \times \vec{c}|} \tag{4}
\]

The intersection test ray/light line is \( d \leq \text{eps} \), \( \text{eps} \) being a predefined tolerance. Let us consider a sample of points \( R \) on the surface as can be seen in Figure 9. We trace back the rays \( AR \) from the eye point \( A \) to the sample points \( R \) and check if the reflected rays \( \vec{RP} \) hit one of the light lines in the light plane. After performing the intersection test for all points \( R \), a sample of reflection points on the reflection lines can be traced.

Texture and Environment Mapping

This is a novel approach based on graphics features of recently developed hardware and software. Reflection lines are simulated by mapping a texture of light tubes on an environment sphere which is then projected onto the surface. The reflection lines are not physically accurate. However, they are
useful as a surface interrogation tool revealing irregularities of the surface. An advantage of this approach is the possibility of updating and real time display of the reflection lines at interactive rates during manipulations of the surface such as rotations, zooms etc.

4 Reflection Lines as Countours

The main result in paper [16] reveals that reflection lines turn out to be related to contour lines of a certain nonlinear function $f$. The function $f$ is defined using an original interpretation of some equations derived from the reflection equation.

**Theorem**

The reflection line of the light line $x = c$ situated in the light plane $z = z_0$ is the image of the mapping of the implicit curve $f(u, v) = c$ from the parameter domain of the surface to the surface. The function $f(u, v)$ is defined by:

$$f(u, v) = \frac{[\vec{s} \cdot \vec{r} \cdot \vec{j}] + z_0 \cdot (\vec{s} \cdot \vec{r})}{\vec{s} \cdot \vec{k}}$$

where $\vec{r} = \vec{r}(u, v)$ is the parametrization of the surface and

$$\vec{s} = \vec{s}(u, v) = 2 \frac{\vec{a} \cdot \vec{n}}{|\vec{a}| |\vec{n}|} \frac{\vec{n}}{|\vec{n}|} - \frac{\vec{a}}{|\vec{a}|}$$

$\diamond$
The theorem is illustrated in Figures 10 and 11. The contours and the reflection lines are marked with black points.

Figure 10: Reflection Lines as Contours in 2D

Figure 11: Reflection Lines on the Surface in 3D

Note that four of the five reflection lines split into two parts.
Remarks

1. This result provides a new way of understanding reflection lines as **global** objects, namely as curves on surfaces defined by mapping of contour curves of a certain nonlinear function $f(u, v)$ from the $(u, v)$ parameter domain into 3D space.

2. Previous approaches based on nonlinear optimization and raytracing provided only **discrete** models of point samplings of reflection lines.

3. Due to this result, the computation and rendering of reflection lines can be performed using standard contouring algorithms and computer graphics software.

5 Conclusion

Reflection lines are important surface interrogation tools in car body design. Several geometric models have been developed for this concept. These models are suited for different computation and rendering techniques in a computer graphics environment. The need of new concepts and algorithms is also driven by needs of the graphics hard- and software companies. Raytracing and texture mapping are well suited for the simulation of reflection lines at interactive rates. The approach based on the insight that reflection lines are related to a well understood geometric object, namely contour curves is particularly useful for a deeper analysis of their geometric features.

Figure 12: Virtual prototyping. Image courtesy of Dassault Systems.
References


