Lösungen „Grundrechenarten":

Aufgabe 1:

(a) $\frac{2r}{11r}\cdot\frac{55r}{4} = \frac{55}{11}\cdot\frac{2}{4}\cdot\frac{r\cdot r}{r} = \underline{\underline{\frac{5r}{2}}}$

(b) $\frac{27pq}{10q}\cdot\frac{15q}{21p} = \frac{27\cdot 15}{10\cdot 21}\cdot\frac{q\cdot q}{q}\cdot\frac{p}{p} = \underline{\underline{\frac{27q}{14}}}$

(c) $\frac{4x}{3x+6}\cdot\frac{x+2}{2x^2} = \frac{x+2}{3(x+2)}\cdot\frac{4x}{2x^2} = \underline{\underline{\frac{2}{3x}}}$

(d) $\frac{3pq-p^2}{q}\cdot\frac{1}{p-3q} = \frac{p(3q-p)}{-q\cdot(3q-p)} = \underline{\underline{-\frac{p}{q}}}$

(e) $\frac{4xy}{27z^2}\cdot\frac{9z}{16x^2} = \frac{4\cdot 9}{27\cdot 16}\cdot\frac{xyz}{x^2z^2} = \underline{\underline{\frac{1}{12}\cdot\frac{y}{xz}}}$

(f) $\frac{2s-5}{12+6s}\cdot\frac{6+3s}{4s^2-25} = \frac{2s-5}{6(s+2)}\cdot\frac{3(s+2)}{(2s+5)(2s-5)} = \underline{\underline{\frac{1}{2}\cdot\frac{1}{2s+5}}}$

(g) $\frac{a^2-9}{2-3a}\cdot\frac{2a-3a^2}{15+5a} = \frac{(a+3)(a-3)}{2-3a}\cdot\frac{a(2-3a)}{5(a+3)} = \underline{\underline{\frac{a(a-3)}{5}}}$

(h) $\frac{x^2y}{16x-12y}\cdot\frac{16x^2-9y^2}{5x^2y^2} = \frac{x^2y}{4(4x-3y)}\cdot\frac{(4x-3y)(4x+3y)}{5x^2y^2} =$

$= \underline{\underline{\frac{4x+3y}{20y}}}$

Aufgabe 2:

(a) $\frac{\frac{z}{9xy}}{\frac{2z^2}{27x^2y}} = \frac{z}{9xy}\cdot\frac{27x^2y}{2z^2} = \underline{\underline{\frac{3}{2}\cdot\frac{x}{z}}}$

(b) $\frac{\frac{7c^2d}{4x^2y}}{\frac{28cd}{12xy^2}} = \frac{7c^2d}{4x^2y}\cdot\frac{12xy^2}{28cd} = \underline{\underline{\frac{3}{4}\cdot\frac{cy}{x}}}$

(c) $\frac{\frac{5a+10b}{9ab}}{\frac{2ab+4b^2}{15a^2b}} = \frac{\frac{5(a+2b)}{9ab}}{\frac{2b(a+2b)}{15a^2b}} = \frac{5(a+2b)}{9ab}\cdot\frac{15a^2b}{2b(a+2b)} = \underline{\underline{\frac{25a}{6b}}}$

(d) $\dfrac{\frac{5p^2q}{2p-6q}}{\frac{10pq^2}{p^2-9q^2}} = \dfrac{5p^2q}{2(p-3q)} \cdot \dfrac{(p-3q)(p+3q)}{10pq^2} = \underline{\underline{\dfrac{1}{4}\cdot\dfrac{p}{q}\cdot(p+3q)}}$

Aufgabe 3:

(a) $\dfrac{5}{6b} + \dfrac{2}{3b} = \dfrac{5+2\cdot 2}{6b} = \dfrac{9}{6b} = \underline{\underline{\dfrac{3}{2b}}}$

(b) $\dfrac{a}{2} - \dfrac{3}{4a} = \underline{\underline{\dfrac{2a^2-3}{4a}}}$

(c) $\dfrac{2}{rs} - \dfrac{1}{r} + \dfrac{1}{s} = \underline{\underline{\dfrac{2-s+r}{rs}}}$

(d) $\dfrac{1}{a^2} + \dfrac{1}{b^2} - \dfrac{2}{ab} = \dfrac{b^2+a^2-2ab}{a^2b^2} = \underline{\underline{\dfrac{(a-b)^2}{(ab)^2}}}$

(e) $\dfrac{7}{p} + \dfrac{4p-7q}{pq} = \dfrac{7q+4p-7q}{pq} = \underline{\underline{\dfrac{4}{q}}}$

(f) $\dfrac{1}{2x} + \dfrac{x-2y}{4xy} = \dfrac{2y+x-2y}{4xy} = \underline{\underline{\dfrac{1}{4y}}}$

(g) $\dfrac{1+mn}{m^2n} - \dfrac{n+1}{mn} = \dfrac{1+mn-m(n+1)}{m^2n} = \underline{\underline{\dfrac{1-m}{m^2n}}}$

(h) $\dfrac{x}{3} - \dfrac{2x}{5} + \dfrac{7x}{15} - \dfrac{x}{9} = x\cdot\left(\dfrac{15-18+21-5}{45}\right) = \underline{\underline{\dfrac{13}{45}\cdot x}}$

(i) $\dfrac{m+2n^2}{7mn^2} - \dfrac{6+6m}{21m} + \dfrac{1+12m^2}{42m^2} =$

$= \dfrac{6m(m+2n^2) - 2mn^2\cdot 6(1+m) + n^2(1+12m^2)}{42m^2n^2}$

$= \dfrac{6m^2+12mn^2-12mn^2-12m^2n^2+n^2+12m^2n^2}{42m^2n^2}$

$= \underline{\underline{\dfrac{6m^2+n^2}{42m^2n^2}}}$

(j) $\dfrac{b}{a^2+ab} + \dfrac{a-b}{(a+b)^2} = \dfrac{b}{a(a+b)} + \dfrac{a-b}{(a+b)(a+b)} =$

$= \dfrac{b(a+b)+a(a-b)}{a(a+b)^2} = \underline{\underline{\dfrac{a^2+b^2}{a(a+b)^2}}}$

(k) $\frac{a^2+b^2}{2a^2-2b^2} - \frac{a-b}{4a+4b} + \frac{b+a}{4a-4b} =$

$= \frac{a^2+b^2}{2(a+b)(a-b)} - \frac{a-b}{4(a+b)} + \frac{a+b}{4(a-b)} =$

$= \frac{2(a^2+b^2) - (a-b)(a-b) + (a+b)(a+b)}{4(a-b)(a+b)} =$

$= \frac{2a^2+2b^2+4ab}{4(a-b)(a+b)} = \frac{2(a+b)^2}{4(a+b)(a-b)} = \underline{\underline{\frac{a+b}{2(a-b)}}}$

(l) $\frac{1+y}{x+y} + \frac{1+x}{x+y} - \frac{2xy}{x^2+y^2+2xy} =$

$= \frac{(x+y)(1+y) + (x+y)(1+x) - 2xy}{(x+y)^2} =$

$= \frac{x+y+xy+y^2 + x+y+x^2+xy - 2xy}{(x+y)^2} =$

$= \underline{\underline{\frac{x^2+y^2+2x+2y}{(x+y)^2}}}$

(m) $\frac{x}{x+5} + x+5 - \frac{x^3+16x^2+55x}{x^2+10x+25} =$

$= \frac{(x+5)x + (x+5)^2(x+5) - x^3-16x^2-55x}{(x+5)^2} =$

$= \frac{x^2+5x + x^3+15x^2+75x+125 - x^3-16x^2-55x}{(x+5)^2} =$

$= \frac{25x+125}{(x+5)^2} = \frac{25(x+5)}{(x+5)^2} = \underline{\underline{\frac{25}{x+5}}}$

Aufgabe 4:

(a) $\left(\frac{y}{3x} - \frac{2y}{5x} + \frac{y}{15}\right) \cdot \frac{15x}{y} = \frac{5y-6y+xy}{15x} \cdot \frac{15x}{y} =$

$= \frac{-y+xy}{y} = \underline{\underline{x-1}}$

(b) $\left(\frac{7}{12ab} + \frac{2a}{3b} - \frac{3b}{4a}\right) \cdot \frac{24a^2b}{11} =$

$$= \frac{7 + 8a^2 - 9b^2}{12ab} \cdot \frac{24a^2b}{11} = \underline{\underline{\frac{(7+8a^2-9b^2)\cdot 2a}{11}}}$$

(c) $\dfrac{\frac{5b^2}{16y} - \frac{25b^2}{12y^2}}{\frac{25b}{6y}} = \frac{5b^2}{4y}\left(\frac{1}{4} - \frac{5}{3y}\right) \cdot \frac{6y}{25b} =$

$$= \frac{3b}{10} \cdot \frac{3y-20}{12y} = \underline{\underline{\frac{b}{40y} \cdot (3y-20)}}$$

(d) $\dfrac{\frac{4m^2}{22p}}{\frac{34m}{55p} \cdot \frac{28m}{51p^2}} = \frac{4m^2}{22p} \cdot \frac{55p}{34m} \cdot \frac{51p^2}{28m} =$

$$= \frac{\cancel{4}\cdot 5 \cdot \cancel{11} \cdot 3 \cdot \cancel{17}}{2 \cdot \cancel{11} \cdot 2 \cdot \cancel{17} \cdot \cancel{4} \cdot 7} \cdot p^2 = \underline{\underline{\frac{15}{28} p^2}}$$

(e) $\dfrac{\frac{a-2}{a}}{\frac{a^2-4}{4a^3}} \cdot \frac{a+2}{2-a} = \frac{a-2}{a} \cdot \frac{4a^3}{(a+2)(a-2)} \cdot \frac{a+2}{a-2} \cdot (-1)$

$$= \underline{\underline{-\frac{4a^2}{a-2}}}$$

<u>Aufgabe 5 :</u>

(a) $\left(1 + \frac{s^2}{r^2}\right)\left(\frac{1}{r^2-s^2} - \frac{1}{r^2+s^2}\right) - \frac{2}{r^2-s^2} =$

$$= \frac{\cancel{r^2+s^2}}{r^2} \cdot \frac{r^2+s^2-(r^2-s^2)}{(r^2-s^2)(\cancel{r^2+s^2})} - \frac{2}{r^2-s^2} =$$

$$= \frac{1}{r^2-s^2} \cdot \left(\frac{2s^2}{r^2} - 2\right) = \frac{1}{r^2-s^2} \cdot \frac{2(s^2-r^2)}{r^2} = \underline{\underline{-\frac{2}{r^2}}}$$

(b) $\left(\frac{a}{b} + \frac{a}{c} - \frac{bc}{a}\right)^2 - \left(\frac{a}{bc}\right)^2 \cdot (b+c)^2 - \left(\frac{bc}{a} - \frac{a}{b}\right)^2 + \frac{a^2}{b^2} =$

$$= \left[\left(\frac{a}{b} - \frac{bc}{a}\right) + \frac{a}{c}\right]^2 - \left(\frac{a}{b}\right)^2 \left(\frac{b+c}{c}\right)^2 - (\ldots)^2 + \left(\frac{a}{b}\right)^2 =$$

$$= \cancel{\left(\frac{a}{b} - \frac{bc}{a}\right)^2} + \frac{2a}{c}\left(\frac{a}{b} - \frac{bc}{a}\right) + \left(\frac{a}{c}\right)^2$$

$$- \cancel{\left(\frac{bc}{a} - \frac{a}{b}\right)^2} - \left(\frac{a}{b}\right)^2\left(\frac{b+c}{c}\right)^2 + \left(\frac{a}{b}\right)^2 =$$

$$= \frac{2a^2}{bc} - 2b + \left(\frac{a}{c}\right)^2 - \left(\frac{a}{b}\right)^2 \cdot \left(\left(\frac{b}{c}+1\right)^2 - 1 \right) =$$

$$= \frac{2a^2}{bc} - 2b + \left(\frac{a}{c}\right)^2 - \left(\frac{a}{b}\right)^2 \cdot \left(\frac{b^2}{c^2} + \frac{2b}{c} \right) =$$

$$= \cancel{\frac{2a^2}{bc}} - 2b + \cancel{\frac{a^2}{c^2}} - \cancel{\frac{a^2}{c^2}} - \cancel{\frac{2a^2}{bc}} = \underline{\underline{-2b}} \quad \text{(uff!)}$$

(c) $$\frac{1}{u} + \frac{1}{v} - \frac{1}{u+v} - \frac{v}{u^2+uv} =$$

$$= \frac{v(u+v) + u(u+v) - uv - v^2}{uv(u+v)} =$$

$$= \frac{u^2 + 2uv + v^2 - uv - v^2}{uv(u+v)} = \frac{u^2+uv}{uv(u+v)} = \underline{\underline{\frac{1}{v}}}$$

Aufgabe 6:

(a) $\mathbb{D} = \mathbb{R} \setminus \{-1/2\}$;

$$\frac{2}{2x+1} = 6 \iff 2 = 12x + 6 \iff \underline{\underline{x = -\frac{1}{3}}}$$

(b) $\mathbb{D} = \mathbb{R} \setminus \{7/2\}$;

$$\frac{6y+3}{7-2y} = \frac{1}{2} \iff 12y + 6 = 7 - 2y \iff \underline{\underline{y = \frac{1}{14}}}$$

(c) $\mathbb{D} = \mathbb{R} \setminus \{-1\}$;

$$\frac{12}{5} = \frac{2+x}{2x+2} \iff 24x + 24 = 10 + 5x \iff \underline{\underline{x = -\frac{14}{19}}}$$

(d) $\mathbb{D} = \mathbb{R} \setminus \{\pm 3\}$;

$$\frac{7}{x+3} = \frac{5}{x-3} \iff 7x - 21 = 5x + 15 \iff \underline{\underline{x = 18}}$$

(e) $\mathbb{D} = \mathbb{R}\setminus\{0; 9/4\}$;

$$\frac{-3+4x}{8x-18} = \frac{5x+6}{10x} \Leftrightarrow -30x+40x^2 = 40x^2-42x+108$$
$$\Leftrightarrow \underline{\underline{x=-9}}$$

(f) $\mathbb{D} = \mathbb{R}\setminus\{0; -5/2\}$;

$$\frac{9x+6}{2x+5} = \frac{9x-5}{2x} \Leftrightarrow 18x^2+12x = 18x^2+35x-25$$
$$\Leftrightarrow \underline{\underline{x = \frac{25}{23}}}$$

(g) $\mathbb{D} = \mathbb{R}\setminus\{-4; 3\}$;

$$\frac{3y+4}{y+4} = 5+\frac{5-2y}{y-3} = \frac{3y-10}{y-3} \Leftrightarrow$$
$$\Leftrightarrow 3y^2-5y-12 = 3y^2+2y-40 \Leftrightarrow$$
$$\Leftrightarrow 7y = 28 \Leftrightarrow \underline{\underline{y=4}}$$

(h) $\mathbb{D} = \mathbb{R}\setminus\{\pm 1; 2\}$;

$$\frac{1}{x-2} - \frac{2}{x-1} + \frac{1}{x+1} = 0 \Leftrightarrow$$
$$\Leftrightarrow \frac{x^2-1 - 2(x-2)(x+1) + (x-2)(x-1)}{(x-2)(x^2-1)} = 0$$
$$\Leftrightarrow x^2-1-2x^2+2x+4+x^2-3x+2 = 0$$
$$\Leftrightarrow \underline{\underline{x=5}}$$

(i) $\mathbb{D} = \mathbb{R}\setminus\{0; 1; 2\}$;

$$\frac{12}{x} - \frac{1}{x-1} - \frac{11}{x-2} = 0 \Leftrightarrow$$
$$\Leftrightarrow 12(x-1)(x-2) - x(x-2) - 11x(x-1) = 0$$
$$\Leftrightarrow 12x^2-36x+24-x^2+2x-11x^2+11x = 0$$
$$\Leftrightarrow \underline{\underline{x = \frac{24}{23}}}$$

(j) $\mathbb{D} = \mathbb{R} \setminus \{5; -3\}$;

$$\frac{x^2-4}{x-5} - \frac{x^2+4}{x+3} = 8$$

$$\Leftrightarrow (x^2-4)(x+3) - (x^2+4)(x-5) = 8(x-5)(x+3)$$

$$\Leftrightarrow \cancel{x^3} + \cancel{3x^2} - 4x - 12 - \cancel{x^3} + \cancel{5x^2} - 4x + 20 = \cancel{8x^2} - 16x - 120$$

$$\Leftrightarrow 8x = -128 \Leftrightarrow \underline{\underline{x = -16}}$$

Aufgabe 7:

(a) $(a-b)^2 + (a+b)^2 = \underline{\underline{2(a^2+b^2)}}$

(b) $(x-y)(x+y) + (x-y)^2 = x^2 - y^2 + x^2 - 2xy + y^2 =$

$$= 2x^2 - 2xy = \underline{\underline{2x(x-y)}}$$

(c) $(u-3w)^2 - (u+3w)^2 = u^2 - 6uw + 9w^2 - (u^2 + 6uw + 9w^2) = \underline{\underline{-12uw}}$

(d) $(a+b+c)^2 - (a+b-c)^2 = ([a+b]+c)^2 - ([a+b]-c)^2$

$$= \underline{\underline{4 \cdot [a+b] \cdot c}}$$

(e) $(a+b+c)^2 - (a+b-c)^2 + (a-b+c)^2 - (a-b-c)^2$

$$= ([a+b]+c)^2 - ([a+b]-c)^2 + ([a-b]+c)^2 - ([a-b]-c)^2$$

$$= 4 \cdot [a+b]c + 4 \cdot [a-b]c = \underline{\underline{8ac}}$$

(f) $(x-y^3)^2 + (x+y^3)^2 = 2x^2 + 2y^6 = \underline{\underline{2(x^2+y^6)}}$

(g) $(x^2+5)^2 \cdot (5-x^2) = (5+x^2)^2 \cdot (5-x^2)$

$$= (5+x^2) \cdot (5+x^2) \cdot (5-x^2) = (5+x^2)(25-x^4)$$

$$= \underline{\underline{125 + 25x^2 - 5x^4 - x^6}}$$

(h) $(1+x+x^2+x^3) \cdot (x-1) = x + x^2 + x^3 + x^4 - 1 - x - x^2 - x^3 = \underline{\underline{x^4 - 1}}$

(i) Terme fallen analog (h) paarweise weg

$$\Rightarrow (1+x+\ldots+x^n)(x-1) = \underline{\underline{x^{n+1} - 1}}.$$

(j) $\frac{(x+y)^2+(x-y)^2}{(x+y)^2-(x-y)^2} = \frac{2(x^2+y^2)}{4xy} = \underline{\underline{\frac{x^2+y^2}{2xy}}}$

(k) $\frac{x^2-y^2}{x+y}+\frac{x^2+y^2}{x-y} = x-y+\frac{x^2+y^2}{x-y}$

$= \frac{(x-y)^2+x^2+y^2}{x-y} = \underline{\underline{\frac{2(x^2-xy+y^2)}{x-y}}}$

(l) $\frac{(4r+7s)^5}{(r^2-s^2)^2} : \frac{(4r+7s)^4}{(r-s)^2} =$

$= \frac{(4r+7s)^5}{(r-s)^2(r+s)^2}\cdot\frac{(r-s)^2}{(4r+7s)^4} = \underline{\underline{\frac{4r+7s}{(r+s)^2}}}$

(m) $\left(\sqrt{\frac{1}{6}-z}-\sqrt{z+\frac{1}{6}}\right)^2 \underset{|z|\le 1/6}{=} \frac{1}{3}-2\sqrt{\left(\frac{1}{6}-z\right)\left(\frac{1}{6}+z\right)}$

$= \underline{\underline{\frac{1}{3}\left(1-\sqrt{1-36z^2}\right)}}$

Aufgabe 8 :

(a) $5x^2+20x-105 = 5(x^2+4x-21) = \underline{\underline{5(x+7)(x-3)}}$

(b) $4x^3+28x^2+48x = 4x(x^2+7x+12) = \underline{\underline{4x(x+3)(x+4)}}$

(c) $3a(x-y)-2b(x-y) = \underline{\underline{(3a-2b)(x-y)}}$

(d) $6uw-14mw+v(7m-3u) = 2w(3u-7m)+v(7m-3u)$

$= \underline{\underline{(v-2w)(7m-3u)}}$

(e) $2a(3a+b)-3ab-b^2 = 2a(3a+b)-b(3a+b) = \underline{\underline{(2a-b)(3a+b)}}$

(f) $28x^4+28x^2y+7y^2 = 7(4x^4+4x^2y+y^2) = \underline{\underline{7(2x^2+y)^2}}$

Aufgabe 9 :

(a) $\sqrt{\sqrt{\sqrt{y}}} = ((y^{1/2})^{1/2})^{1/2} = (y^{1/4})^{1/2} = \underline{\underline{y^{1/8}}}$

(b) $\sqrt[4]{b\sqrt[6]{b^2}} = (b\cdot b^{2/6})^{1/4} = (b^{1+1/3})^{1/4} = \underline{\underline{b^{1/3}}}$

(c) $(\sqrt[n]{p^k})^{2n} = ((p^k)^{1/n})^{2n} = (p^{k/n})^{2n} = \underline{\underline{p^{2k}}}$

(d) $\sqrt{\sqrt[7]{36(p+q)^8}} = ((36\cdot(p+q)^8)^{1/7})^{1/2} = (36^{1/7}\cdot(p+q)^{8/7})^{1/2}$

$= 36^{1/7\cdot 1/2}\cdot(p+q)^{4/7} = \underline{\underline{6^{1/7}\cdot(p+q)^{4/7}}}$

(e) $\frac{\sqrt[n]{p^4}}{\sqrt[n]{p}} = \frac{(p^4)^{1/n}}{p^{1/n}} = p^{4/n}\cdot p^{-1/n} = p^{4/n-1/n} = \underline{\underline{p^{3/n}}}$

(f) $\frac{\sqrt[n]{5^k}}{\sqrt{5^{-k}}} = \frac{5^{k/n}}{5^{-k/2}} = 5^{k/n}\cdot 5^{k/2} = \underline{\underline{5^{(k\cdot\frac{2+n}{2n})}}}$

(g) $\sqrt[6]{u^7\cdot\sqrt[3]{u^2v}\cdot\sqrt{v}} = (u^7\cdot(u^2v)^{1/3}\cdot v^{1/2})^{1/6} = (u^7\cdot u^{2/3}\cdot v^{1/3}\cdot v^{1/2})^{1/6}$

$= (u^{23/3}\cdot v^{5/6})^{1/6} = \underline{\underline{u^{23/18}\cdot v^{5/36}}}$

(h) $\sqrt{r^{1/2}\cdot s^{1/2}\cdot\sqrt{st}} = (r^{1/2}s^{1/2}(st)^{1/2})^{1/2} = \underline{\underline{r^{1/4}s^{1/2}t^{1/4}}}$

Aufgabe 10:

(a) $\frac{a^{-3}b^6c}{x^{-2}y^{-1}}\cdot\frac{x^{-3}y^{-2}}{a^{-4}b^5c^{-1}} = a^{-3}b^6c\,x^2y\,x^{-3}y^{-2}a^4b^{-5}c = abc^2x^{-1}y^{-1}$

$= \underline{\underline{\frac{abc^2}{xy}}}$

(b) $\frac{(a^{-3}b)^2}{(x^2y^3)^{-1}}\cdot\frac{x^{-1}y^{-2}}{(a^{-3}b)^{-2}} = \frac{b^2}{a^6}\cdot x^2y^3\cdot\frac{1}{xy^2}\cdot\frac{b^2}{a^6} = \underline{\underline{\frac{b^4xy}{a^{12}}}}$

(c) $\frac{(a^2-b^2)^{-2}}{(a+b)^{-3}}\cdot\frac{(a-b)^2}{a+b} = \frac{(a+b)^3}{((a-b)(a+b))^2}\cdot\frac{(a-b)^2}{a+b} = \underline{\underline{1}}$

(d) $(a^{-1}+b^{-1})^2 = (\frac{1}{a}+\frac{1}{b})^2 = \underline{\underline{(\frac{a+b}{ab})^2}}$

(e) $(a^{-1}+b^{-1})^{-2} = \frac{1}{(a^{-1}+b^{-1})^2} \overset{(d)}{=} \underline{\underline{(\frac{ab}{a+b})^2}}$

(f) $\frac{4^n - 2\cdot 4^{n-1} + 4^{n-2}}{5\cdot 4^{n+1} + 4^{n+2}} = \frac{4^{n-2}\cdot(4^2-2\cdot4+1)}{4^{n+1}\cdot(5+4)} = \underline{\underline{\frac{1}{64}}}$

(g) $\frac{\sqrt{3}-3}{2\sqrt{3}} - \frac{\sqrt{3}}{1+\sqrt{3}} = \frac{(\sqrt{3}-3)(1+\sqrt{3}) - 2\cdot\sqrt{3}\sqrt{3}}{2\cdot\sqrt{3}\cdot(1+\sqrt{3})} =$

$= \frac{\sqrt{3}-3\sqrt{3}-6}{2\sqrt{3}+6} = \underline{\underline{-1}}$

(h) $\frac{z^{2+n}-z^n}{z^{n+1}+z^n} = \frac{z^n\cdot(z^2-1)}{z^n\cdot(z+1)} = \underline{\underline{z-1}}$

(i) $(\frac{2}{z^2}-\frac{1}{z^{-2}})^2 = (\frac{2}{z^2}-z^2)^2 = \underline{\underline{\frac{4}{z^4}-4+z^4}}$

(j) $$\sqrt{n} + \frac{1}{\sqrt{n}+\sqrt{n+1}} = \sqrt{n} + \frac{\sqrt{n}-\sqrt{n+1}}{(\sqrt{n}+\sqrt{n+1})(\sqrt{n}-\sqrt{n+1})} =$$

$$= \sqrt{n} + \frac{\sqrt{n}-\sqrt{n+1}}{-1} = \frac{-\sqrt{n}+\sqrt{n}-\sqrt{n+1}}{-1} = \underline{\underline{\sqrt{n+1}}}$$

(k) $$\frac{x\sqrt{x}+x\sqrt{y}}{x-y} - \sqrt{x} - \frac{\sqrt{xy}}{\sqrt{x}-\sqrt{y}} =$$

$$= \frac{x(\sqrt{x}+\sqrt{y}) - \sqrt{x}(x-y) - \sqrt{xy}(\sqrt{x}+\sqrt{y})}{x-y} =$$

$$= \frac{(x-\sqrt{xy})(\sqrt{x}+\sqrt{y}) - \sqrt{x}(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{x-y}$$

$$= \frac{\sqrt{x}(\sqrt{x}-\sqrt{y}) - \sqrt{x}(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}} = \underline{\underline{0}}$$

Aufgabe 11:

(a) $$\sqrt{\frac{1}{4}x^3y + \frac{1}{2}x^2y^2 + \frac{1}{4}xy^3} = \sqrt{\frac{1}{4}xy(x^2+2xy+y^2)}$$

$$= \underline{\underline{\frac{1}{2}\sqrt{xy}\cdot|x+y|}} \quad (!)$$

(b) $$\left(\frac{e^x-e^{-x}}{2}\right)^2 + \left(\frac{e^x+e^{-x}}{2}\right) = \frac{e^{2x}-2+e^{-2x}+e^{2x}+2+e^{-2x}}{4}$$

$$= \underline{\underline{\frac{e^{2x}+e^{-2x}}{2}}}$$

(c) $$2^{-1}\cdot\left[\left(\frac{1}{2}a^{-4}+a^4\right)^2 - \left(\frac{1}{2}a^{-4}-a^4\right)^2\right] =$$

$$= \frac{1}{2}\cdot\left[\frac{a^{-8}}{4} + 1 + a^8 - \frac{a^{-8}}{4} + 1 - a^8\right] = \underline{\underline{1}}$$

(d) $$(r^x)^y\cdot(r^y)^x\cdot((r^x)^y+(r^y)^x) = r^{xy}\cdot r^{yx}\cdot(r^{xy}+r^{yx})$$

$$= r^{2xy}\cdot 2r^{xy} = \underline{\underline{2\cdot r^{3xy}}}$$

(e) $$(\sqrt{a^3b}-a)^2 - (\sqrt{a^5b}+a)(\sqrt{ab}+a) =$$

$$= (\cancel{a^3b} - 2a^2\sqrt{ab} + \cancel{a^2}) - (\cancel{a^3b} + a^3\sqrt{ab} + a\sqrt{ab} + \cancel{a^2})$$

$$= -2a^2\sqrt{ab} - a^3\sqrt{ab} - a\sqrt{ab} = -a\sqrt{ab}(a^2+2a+1)$$

$$= \underline{\underline{-a(a+1)^2\sqrt{ab}}}.$$

(f) $\left(\sqrt{2a}-\sqrt{3b}\right)^2-\left(\sqrt{3a}-\sqrt{2b}\right)^2 =$

$= 2a - \cancel{2\sqrt{6ab}} + 3b - \left(3a - \cancel{2\sqrt{6ab}} + 2b\right) = \underline{\underline{-a+b}}$

(g) $\left(\sqrt{a-b}+3\sqrt{a+b}\right)\left(\sqrt{a-b}-3\sqrt{a+b}\right) =$

$= a-b-9(a+b) = \underline{\underline{-8a-10b}}$

(h) $\left(\sqrt{\frac{2a^2}{b^3}}-\sqrt{\frac{2b^3}{a^2}}\right)^2-\left(b\sqrt{\frac{3b}{a^2}}-\frac{1}{b}\sqrt{\frac{3a^2}{b}}\right)^2 =$

$= \frac{2a^2}{b^3} - 4 + \frac{2b^3}{a^2} - \left(\frac{3b^3}{a^2} - 6 + \frac{3a^2}{b^3}\right) =$

$= -\frac{a^2}{b^3} + 2 - \frac{b^3}{a^2} = -\frac{a^4-2a^2b^3+b^6}{a^2b^3} =$

$= \underline{\underline{-\frac{(a^2-b^3)^2}{a^2b^3}}}$

Aufgabe 12:

(a) $2 = \sqrt{x}\cdot x^{1/3} = x^{1/2}\cdot x^{1/3} = x^{5/6} \Rightarrow \underline{\underline{x = 2^{6/5}}}.$

Probe:

$\sqrt{2^{6/5}}\cdot\left(2^{6/5}\right)^{1/3} = \left(2^{6/5}\right)^{1/2}\cdot\left(2^{6/5}\right)^{1/3} = 2^{6/5\cdot 1/2}\cdot 2^{6/5\cdot 1/3}$

$= 2^{3/5}\cdot 2^{2/5} = 2^{3/5+2/5} = 2 \quad ✓$

(b) $\sqrt[3]{x-1} = \sqrt[6]{x+5} \Leftrightarrow (x-1)^{1/3} = (x+5)^{1/6}$

$\Rightarrow (x-1)^2 = x+5 \Leftrightarrow x^2-3x-4=0\ ;$ $(x_1=-1)$ $\underline{\underline{x_2=4}}$

Probe:

$\sqrt[3]{4-1} \stackrel{?}{=} \sqrt[6]{9} \Leftrightarrow 3^{1/3} = (3^2)^{1/6} \quad ✓$

(c) $\sqrt[8]{x-1}-\sqrt[4]{x-1} = 0 \Leftrightarrow (x-1)^{1/8} = (x-1)^{1/4}$

$\Rightarrow x-1 = (x-1)^2 \Leftrightarrow x^2-3x+2=0\ ;$ $\underline{\underline{x_1=1}}$ $\underline{\underline{x_2=2}}$

Probe:

- $\sqrt[8]{1-1}-\sqrt[4]{1-1} = 0 \quad ✓$
- $\sqrt[8]{2-1}-\sqrt[4]{2-1} = 0 \quad ✓$

(d) $\sqrt{x+1} + \sqrt{x} = 2 \qquad (x \geq 0\,!)$

$$\Rightarrow \quad 4 = \left(\sqrt{x+1} + \sqrt{x}\right)^2 = 2x + 1 + 2\sqrt{x}\sqrt{x+1}$$

$$\Leftrightarrow \quad 3 - 2x = 2\sqrt{x}\sqrt{x+1}$$

$$\Rightarrow \quad (3-2x)^2 = 4x(x+1) \quad \Leftrightarrow \quad -16x = -9, \ \underline{\underline{x = \frac{9}{16}}}$$

Probe:

$$\sqrt{\frac{9}{16} + 1} + \sqrt{\frac{9}{16}} = \frac{5}{4} + \frac{3}{4} = 2 \quad \checkmark$$

(e) $\frac{1}{(x-2)^{-1/2}} = 5 - \sqrt{x-3} \qquad (x \geq 3\,!)$

$$\Leftrightarrow \quad \sqrt{x-2} + \sqrt{x-3} = 5$$

$$\Rightarrow \quad 25 = \left(\sqrt{x-2} + \sqrt{x-3}\right)^2 = 2x - 5 + 2\sqrt{x-2}\sqrt{x-3}$$

$$\Leftrightarrow \quad 15 - x = \sqrt{x-2}\sqrt{x-3}$$

$$\Rightarrow \quad (15-x)^2 = (x-2)(x-3) = x^2 - 5x + 6$$

$$\Leftrightarrow \quad -25x = -219 \ ; \quad \underline{\underline{x = \frac{219}{25}}}$$

Probe:

- $\frac{1}{\left(\frac{219}{25} - 2\right)^{-1/2}} = \sqrt{\frac{219-50}{25}} = \frac{13}{5} \longleftarrow = \checkmark$
- $5 - \sqrt{\frac{219}{25} - 3} = 5 - \sqrt{219-75} = 5 - \frac{12}{5} = \frac{13}{5}$

(f) $\frac{1}{3}\sqrt{x} - \sqrt[3]{x} = 0 \quad \Leftrightarrow \quad \frac{1}{3}\sqrt{x} = \sqrt[3]{x} \quad (*)$

Fallunterscheidung:

(1) $x = 0$: $\quad (*) \Leftrightarrow 0 = 0. \quad$ $\underline{\underline{x=0}}$ ist Lösung.

(2) $x \neq 0$: $\quad (*) \Leftrightarrow \frac{1}{3} = x^{1/3} \cdot \frac{1}{\sqrt{x}} = x^{-1/6} \ ; \quad \underline{\underline{x = 3^6 = 729}}$

Probe zu (2):

$$\frac{1}{3} \cdot \sqrt{3^6} - \sqrt[3]{3^6} = \frac{1}{3} \cdot 3^3 - 3^2 = 9 - 9 = 0 \quad \checkmark$$

(g) $3\cdot\sqrt[3]{x^2} - x = \frac{2}{x^{-1/3}} \Rightarrow 3\cdot x^{2/3} - x = 2\cdot x^{1/3}$ (*)

Fallunterscheidung:

(1) $x=0$: (*) lautet „$0=0$"; aber $0 \notin \mathbb{D}$!

(2) $x\neq 0$: Multipliziere (*) mit $x^{-1/3}$:

$(*) \Leftrightarrow 3\cdot x^{1/3} - x^{2/3} = 2.$ Setze $z := x^{1/3}$

$\Leftrightarrow 3z - z^2 = 2;\quad z_1 = 1, z_2 = 2 \Rightarrow \underline{\underline{x_1=1, x_2=8}}.$

Probe zu (2):

- $3\cdot\sqrt[3]{1^2} - 1 \overset{?}{=} \frac{2}{1^{-1/3}} \Leftrightarrow 3-1 = 2$ ✓
- $3\cdot\sqrt[3]{8^2} - 8 \overset{?}{=} \frac{2}{8^{-1/3}} \Leftrightarrow 3\cdot4-8 = 4$ ✓

(h) $3\sqrt{3x+1} + 2 = 0 \Leftrightarrow \underbrace{3\sqrt{3x+1}}_{\geq 0} = -2$ keine Lösung!

(i) $\sqrt[4]{1+\sqrt{x-1}} = \sqrt{2} \Rightarrow$

$\Rightarrow 1+\sqrt{x-1} = 4 \Leftrightarrow \sqrt{x-1} = 3$

$\Rightarrow x-1 = 9;\quad \underline{\underline{x=10}}$

Probe:

$\sqrt[4]{1+\sqrt{10-1}} = \sqrt[4]{1+3} = \sqrt{2}$ ✓

(j) $\sqrt[4]{2\cdot\sqrt{2x+1} + 10} = 2$

$\Rightarrow 2\cdot\sqrt{2x+1} + 10 = 16 \Leftrightarrow \sqrt{2x+1} = 3$

$\Rightarrow 2x+1 = 9;\quad \underline{\underline{x=4}}.$

Probe:

$\sqrt[4]{2\cdot\sqrt{2\cdot4+1} + 10} = \sqrt[4]{2\cdot3+10} = \sqrt[4]{16} = 2$ ✓

Aufgabe 13:

(a) $\log_7 49 = \log_7(7^2) = 2\cdot\log_7 7 = \underline{\underline{2}}$

(b) $\log_{49} 7 = \log_{49}\sqrt{49} = \log_{49}(49^{1/2}) = \frac{1}{2}\log_{49}(49) = \underline{\underline{1/2}}$

(c) $\log_4 32 = \log_4(4\cdot4\cdot\sqrt{4}) = \log_4 4 + \log_4 4 + \frac{1}{2}\log_4 4 = \underline{\underline{5/2}}$

(d) $\log_3(1/9) = \log_3(3^{-2}) = -2\log_3 3 = \underline{\underline{-2}}$

(e) $\log_4 \sqrt{32} = \log_4(32^{1/2}) = \frac{1}{2}\log_4 32 \overset{(c)}{=} \underline{\underline{5/4}}$

(f) $\log_2(2\sqrt{2}) = \log_2(2^{3/2}) = 3/2 \log_2 2 = \underline{\underline{3/2}}$

(g) $\log_5(25^{1/3}) = \log_5(5^{2/3}) = 2/3 \log_5 5 = \underline{\underline{2/3}}$

(h) $\ln(\sqrt{e^{2x+4}}) = 1/2 \cdot \ln(e^{2x+4}) = 1/2(2x+4) = \underline{\underline{x+2}}$

(i) $\log_{\sqrt[4]{x}}(x^4) = \log_{\sqrt[4]{x}}(\sqrt[4]{x}^{16}) = 16\log_{\sqrt[4]{x}}(\sqrt[4]{x}) = \underline{\underline{16}}$

(j) $\log_a(\sqrt[n]{a^m}) = \log_a(a^{m/n}) = m/n \log_a a = \underline{\underline{m/n}}$

(k) $\log_x(1/x^{-2}) = \log_x(x^2) = 2\log_x x = \underline{\underline{2}}$

Aufgabe 14:

(a) $\log_4 128 + \log_3(9\sqrt{3}) = \log_4(4^{7/2}) + \log_3(3^{5/2}) = \frac{7}{2} + \frac{5}{2} = \underline{\underline{6}}$

(b) $\log_2(1/8) + \log_4(1/8) + \log_8(1/8) =$

$= \log_2(2^{-3}) + \log_4(4^{-3/2}) + \log_8(8^{-1}) = -3 - \frac{3}{2} - 1 = \underline{\underline{-\frac{11}{2}}}$

(c) $\log_a 5 + \log_a(10a) - \log_a\sqrt{a} - \log_a 50 =$

$= \log_a\left(\frac{5 \cdot 10a}{\sqrt{a} \cdot 50}\right) = \log_a\sqrt{a} = \frac{1}{2}\log_a a = \underline{\underline{\frac{1}{2}}}$

(d) $\log_2(a^2+6a+9) - \log_2(a+3) = \log_2\left(\frac{a^2+6a+9}{a+3}\right)$

$= \log_2\left(\frac{(a+3)^2}{a+3}\right) = \underline{\underline{\log_2(a+3)}}$

Aufgabe 15:

(a) $\log_2 x = 4 \Leftrightarrow \underline{\underline{x}} = 2^4 = \underline{\underline{16}}$.

(b) $\log_2(x+20) = 5 \Leftrightarrow x+20 = 2^5 = 32 \Leftrightarrow \underline{\underline{x = 12}}$

(c) $\log_{16} x = 3/4 \Leftrightarrow \underline{\underline{x}} = 16^{3/4} = 2^3 = \underline{\underline{8}}$

(d) $\log_x 8 = 3 \Leftrightarrow 8 = x^3 \Leftrightarrow \underline{\underline{x = 2}}$

(e) $\log_x\sqrt{32} = 5 \Leftrightarrow \log_x 32 = 10 \Leftrightarrow 32 = x^{10} \Leftrightarrow \underline{\underline{x}} = \sqrt[10]{32} = \underline{\underline{\sqrt{2}}}$

(f) $\log_x\sqrt{2} = \frac{1}{4} \Leftrightarrow \log_x 2 = \frac{1}{2} \Leftrightarrow 2 = x^{1/2} \Leftrightarrow \underline{\underline{x}} = 2^2 = \underline{\underline{4.}}$

Aufgabe 16 :

(a) $6^{2x} - 6^x - 2 = 0.$ Setze $z := 6^x$, $x = \log_6 z$

$\Leftrightarrow z^2 - z - 2 = 0$; $(z_1 = -1)$, $z_2 = 2$, $\underline{\underline{x = \log_6 2}}$

Probe :

$6^{2\cdot\log_6 2} - 6^{\log_6 2} - 2 = 6^{\log_6 4} - 6^{\log_6 2} - 2 = 4 - 2 - 2 = 0.$ ✓

(b) $25^{x+1} - 25^x - 4 = 0.$ Setze $z := 25^x$, $x = \log_{25} z$

$\Leftrightarrow 25\cdot z - z - 4 = 0 \Leftrightarrow z = \frac{1}{6}$, $\underline{\underline{x = -\log_{25} 6}}.$

Probe :

$25^{-\log_{25}6+1} - 25^{-\log_{25}6} - 4 = 24\cdot 25^{-\log_{25}6} - 4 = \frac{24}{6} - 4 = 0$ ✓

(c) $9^{2x-1/2}\cdot 27^{x+5} = 1 \Leftrightarrow 3^{4x}\cdot\frac{1}{3}\cdot 3^{3x}\cdot 3^{15} = 1$

$\Leftrightarrow 3^{7x} = 3^{-14} \Leftrightarrow \underline{\underline{x = -2}}.$

Probe :

$9^{2\cdot(-2)-1/2}\cdot 27^{(-2)+5} = 9^{-9/2}\cdot 27^3 = 3^{-9}\cdot 3^9 = 1$ ✓

(d) $\left(\frac{1}{\sqrt{2}}\right)^{-2x}\cdot 2^{x-1} = \frac{1}{2} \Leftrightarrow \sqrt{2}^{2x}\cdot 2^{x-1} = 2^{-1}$

$\Leftrightarrow 2^x\cdot 2^{x-1} = 2^{-1} \Leftrightarrow 2^{2x-1} = 2^{-1} \Leftrightarrow \underline{\underline{x = 0}}.$

Probe :

$\left(\frac{1}{\sqrt{2}}\right)^{-2\cdot 0}\cdot 2^{0-1} = 2^{-1}$ ✓

(e) $2^x - 4\cdot 2^{-x} + 3 = 0 \Leftrightarrow 2^{2x} - 4 + 3\cdot 2^x = 0$

Setze $z := 2^x$, $x = \log_2 z$ und erhalte

$z^2 + 3z - 4 = 0$; $(z_1 = -4)$, $z_2 = 1$. $\underline{\underline{x_2 =}} \log_2 1 \underline{\underline{= 0}}.$

Probe :

$2^0 - 4\cdot 2^{-0} + 3 = 1 - 4 + 3 = 0$ ✓

(f) $\lg x - \frac{2}{\lg x} - 1 = 0 \qquad (x>0,\ x\neq 1)$

Multipliziere mit $\lg x$:

$\Rightarrow (\lg x)^2 - 2 - (\lg x) = 0.$ Setze $z := \lg x$, $x = 10^z$.

$\Leftrightarrow z^2 - z - 2 = 0$; $z_1 = -1$, $z_2 = 2$; $\underline{\underline{x_1 = \frac{1}{10},\ x_2 = 100}}$.

Probe:

- $\lg\left(\frac{1}{10}\right) - \frac{2}{\lg\left(\frac{1}{10}\right)} - 1 = -1 - \frac{2}{-1} - 1 = 0$ ✓
- $\lg(100) - \frac{2}{\lg(100)} - 1 = 2 - \frac{2}{2} - 1 = 0$ ✓

(g) $\left(\frac{1}{25}\right)^x + 5^{1-x} = 6 \Leftrightarrow 5^{-2x} + 5\cdot 5^{-x} - 6 = 0$

Setze $z := 5^{-x}$, $x = -\log_5 z$

$\Leftrightarrow z^2 + 5z - 6 = 0$; $z_1 = 1$, $(z_2 = -6)$; $\underline{\underline{x_1 = -\log_5 1 = 0}}$.

Probe:

$\left(\frac{1}{25}\right)^0 + 5^{1-0} = 1 + 5 = 6$ ✓

(h) $\lg(x-2) - \lg(x-4) = 2 \qquad (x>4)$

$\Rightarrow \lg\left(\frac{x-2}{x-4}\right) = 2 \Leftrightarrow \frac{x-2}{x-4} = 10^2 = 100$

$\Leftrightarrow x - 2 = 100x - 400 \Leftrightarrow \underline{\underline{x = \frac{398}{99} = 4 + \frac{2}{99}}}$

Probe:

$\lg\left(4 + \frac{2}{99} - 2\right) - \lg\left(4 + \frac{2}{99} - 4\right) = \lg\left(2 + \frac{2}{99}\right) - \lg\left(\frac{2}{99}\right)$

$= \lg\left(\frac{2 + \frac{2}{99}}{\frac{2}{99}}\right) = \lg\left(\frac{198+2}{2}\right) = \lg(100) = 2.$ ✓

(i) $\lg x + \lg(x+3) = 1 \qquad (x>0)$

$\Rightarrow \lg(x(x+3)) = 1 \Leftrightarrow x(x+3) = 10$

$\Leftrightarrow \underline{\underline{x_1 = 2}}$, $(x_2 = -5)$

Probe:

$\lg 2 + \lg 5 = \lg(2\cdot 5) = \lg 10 = 1$ ✓

(j) $3^x + 3^{-x} = \frac{10}{3} \Leftrightarrow 3^{2x} + 1 = \frac{10}{3} \cdot 3^x$

Setze $z := 3^x$, $x = \log_3 z$

$\Leftrightarrow z^2 + 1 = \frac{10}{3} z$; $z_1 = \frac{1}{3}$, $z_2 = 3$; $\underline{\underline{x_1 = -1, x_2 = 1}}$.

Probe:

- $3^{-1} + 3^{-(-1)} = \frac{1}{3} + 3 = \frac{10}{3}$ ✓
- $3^1 + 3^{-1} = 3 + \frac{1}{3} = \frac{10}{3}$ ✓

(k) $8 \cdot 4^{-x} - 6 \cdot 2^{-x} + 1 = 0 \Leftrightarrow 8 \cdot 2^{-2x} - 6 \cdot 2^{-x} + 1 = 0$.

Setze $z := 2^{-x}$, $x = -\log_2 z$.

$\Leftrightarrow 8z^2 - 6z + 1 = 0$; $z_1 = \frac{1}{2}$, $z_2 = \frac{1}{4}$; $\underline{\underline{x_1 = 1, x_2 = 2}}$.

Probe:

- $8 \cdot 4^{-1} - 6 \cdot 2^{-1} + 1 = 2 - 3 + 1 = 0$ ✓
- $8 \cdot 4^{-2} - 6 \cdot 2^{-2} + 1 = \frac{1}{2} - \frac{3}{2} + 1 = 0$ ✓

(l) $\sqrt{2^x} + \frac{8}{\sqrt{2^x}} = 9 \Leftrightarrow 2^{x/2} + 8 \cdot 2^{-x/2} = 9$

$\Leftrightarrow 2^x + 8 = 9 \cdot 2^{x/2}$. Setze $z := 2^{x/2}$, $x = 2\log_2 z$

$\Leftrightarrow z^2 + 8 = 9z$; $z_1 = 1$, $z_2 = 8$; $\underline{\underline{x_1 = 0, x_2 = 6}}$.

Probe:

- $\sqrt{2^0} + \frac{8}{\sqrt{2^0}} \stackrel{?}{=} 9 \Leftrightarrow 1 + \frac{8}{1} = 9$ ✓
- $\sqrt{2^6} + \frac{8}{\sqrt{2^6}} \stackrel{?}{=} 9 \Leftrightarrow 8 + \frac{8}{8} = 9$ ✓

(m) $\lg x + \lg(2x) + \lg(3x) = \lg 6 \quad (x > 0)$

$\Leftrightarrow \lg(x \cdot 2x \cdot 3x) = \lg 6$

$\Leftrightarrow 6x^3 = 6 \Leftrightarrow \underline{\underline{x = 1.}}$

Probe:

$\lg 1 + \lg(2 \cdot 1) + \lg(3 \cdot 1) = 0 + \lg(2 \cdot 3) = \lg 6$ ✓

(n) Sei $y = a^x$. Logarithmieren:

$$\Leftrightarrow \ln(y) = \ln(a^x) = x \cdot \ln a.$$

Exponentialfunktion anwenden:

$$\Leftrightarrow \underbrace{e^{\ln y}}_{=y} = e^{x \cdot \ln a}. \quad \text{Also} \quad a^x \underset{(s.o.)}{=} y = e^{x \ln a}.$$

Sei jetzt $y = \log_a x \Leftrightarrow a^y = x$.

Logarithmieren:

$$\Leftrightarrow \ln x = \ln(a^y) = y \ln a.$$

$$\Leftrightarrow y = \frac{\ln x}{\ln a}. \quad \text{Also} \quad \log_a x \overset{(s.o.)}{=} y = \frac{\ln x}{\ln a}. \quad ■$$