

A) Grundintegrale

$$1) \int x^a dx = \frac{x^{a+1}}{a+1} \quad (a \neq -1)$$

$$2) \int \frac{1}{x} dx = \ln |x|$$

$$3) \int \frac{dx}{1+x^2} = \arctan x$$

$$4) \int \cos x dx = \sin x$$

$$5) \int \sin x dx = -\cos x$$

$$6) \int \frac{1}{\cos^2 x} dx = \tan x$$

$$7) \int \frac{1}{\sin^2 x} dx = -\cot x$$

$$8) \int e^x dx = e^x$$

$$9) \int a^x dx = \frac{a^x}{\ln a}$$

$$10) \int \cosh x dx = \sinh x$$

$$11) \int \sinh x dx = \cosh x$$

$$12) \int \frac{1}{\cosh^2 x} dx = \tanh x$$

$$13) \int \frac{1}{\sinh^2 x} dx = -\coth x$$

$$14) \int \ln x dx = x \ln x - x$$

B) Rationale Funktionen

$$15) \int (x-b)^a dx = \frac{(x-b)^{a+1}}{a+1} \quad (a \neq -1)$$

$$16) \int \frac{1}{x-a} dx = \ln |x-a|$$

$$17) \int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln \left| \frac{x-a}{x-b} \right| \quad (a \neq b)$$

$$18) \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a}$$

$$19) \int \frac{dx}{(x^2 + a^2)^2} = \frac{x}{2a^2(x^2 + a^2)} + \frac{1}{2a^3} \arctan \frac{x}{a}$$

$$\begin{aligned}
20) \int \frac{dx}{ax^2+bx+c} &= \frac{2}{\sqrt{\Delta}} \arctan \frac{2ax+b}{\sqrt{\Delta}} && \text{für } \Delta > 0 \\
&= \frac{1}{\sqrt{-\Delta}} \ln \left| \frac{2ax+b-\sqrt{-\Delta}}{2ax+b+\sqrt{-\Delta}} \right| && \text{für } \Delta < 0 \\
&= \frac{-2}{2ax+b} && \text{für } \Delta = 0
\end{aligned}$$

$$\text{mit } \Delta = 4ac - b^2$$

$$21) \int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c} \quad (\text{vgl. 20})$$

Bei Integralen über weitere echt gebrochen rationale Funktionen wird auf die Methode der Partialbruchzerlegung verwiesen.

C) Irrationale Funktionen

$$22) \int \sqrt{ax+b} dx = \frac{2}{3a} (ax+b)^{3/2}$$

$$23) \int \frac{1}{\sqrt{ax+b}} dx = \frac{2}{a} \sqrt{ax+b}$$

$$24) \int x\sqrt{ax+b} dx = \frac{2(3ax-2b)}{15a^2} (ax+b)^{3/2}$$

$$25) \int \frac{x}{\sqrt{ax+b}} dx = \frac{2(ax-2b)}{3a^2} \sqrt{ax+b}$$

$$\begin{aligned}
26) \int \frac{dx}{x\sqrt{ax+b}} &= \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| && \text{für } b > 0 \\
&= \frac{2}{\sqrt{-b}} \arctan \sqrt{\frac{ax+b}{-b}} && \text{für } b < 0
\end{aligned}$$

$$27) \int \frac{1}{x} \sqrt{ax+b} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}} \quad (\text{vgl. 26})$$

$$28) \int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \arcsin \frac{x}{a}$$

$$29) \int x \sqrt{a^2-x^2} dx = -\frac{1}{3} (a^2-x^2)^{3/2}$$

$$30) \int \frac{1}{x} \sqrt{a^2-x^2} dx = \sqrt{a^2-x^2} - a \ln \left| \frac{a + \sqrt{a^2-x^2}}{x} \right|$$

$$31) \int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a}$$

$$32) \int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2}$$

$$33) \int \frac{x^2}{\sqrt{a^2-x^2}} dx = -\frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \arcsin \frac{x}{a}$$

$$34) \int \sqrt{a^2+x^2} dx = \frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \ln (x + \sqrt{a^2+x^2})$$

$$35) \int x \sqrt{a^2+x^2} dx = \frac{1}{3} (a^2+x^2)^{3/2}$$

$$36) \int \frac{1}{x} \sqrt{a^2+x^2} dx = \sqrt{a^2+x^2} - a \ln \left| \frac{a + \sqrt{a^2+x^2}}{x} \right|$$

$$37) \int \frac{1}{\sqrt{a^2+x^2}} dx = \ln |x + \sqrt{a^2+x^2}|$$

$$38) \int \frac{x}{\sqrt{a^2+x^2}} dx = \sqrt{a^2+x^2}$$

$$39) \int \frac{x^2}{\sqrt{a^2+x^2}} dx = \frac{x}{2} \sqrt{a^2+x^2} - \frac{a^2}{2} \ln |x + \sqrt{a^2+x^2}|$$

$$40) \int \frac{1}{x \sqrt{a^2+x^2}} dx = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2+x^2}}{x} \right|$$

$$41) \int \frac{1}{x^2 \sqrt{a^2+x^2}} dx = -\frac{\sqrt{a^2+x^2}}{a^2 x}$$

$$42) \int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \ln |x + \sqrt{x^2-a^2}|$$

$$43) \int x \sqrt{x^2-a^2} dx = \frac{1}{3} (x^2-a^2)^{3/2}$$

$$44) \int \frac{1}{x} \sqrt{x^2-a^2} dx = \sqrt{x^2-a^2} - a \arccos \frac{a}{x}$$

$$45) \int \frac{1}{\sqrt{x^2-a^2}} dx = \ln |x + \sqrt{x^2-a^2}|$$

$$46) \int \frac{x}{\sqrt{x^2-a^2}} dx = \sqrt{x^2-a^2}$$

$$47) \int \frac{x^2}{\sqrt{x^2-a^2}} dx = \frac{x}{2} \sqrt{x^2-a^2} + \frac{a^2}{2} \ln |x + \sqrt{x^2-a^2}|$$

$$\begin{aligned}
48) \int \frac{dx}{\sqrt{ax^2+bx+c}} &= \frac{1}{\sqrt{a}} \ln |2\sqrt{a} \sqrt{ax^2+bx+c} + 2ax + b| \quad \text{für } a > 0 \\
&= \frac{1}{\sqrt{a}} \ln |2ax+b| \quad \text{für } a > 0, \Delta = 0 \\
&= \frac{-1}{\sqrt{-a}} \arcsin \frac{2ax+b}{\sqrt{-\Delta}} \quad \text{für } a < 0, \Delta < 0 \\
\text{mit } \Delta &= 4ac - b^2
\end{aligned}$$

$$49) \int \frac{x dx}{\sqrt{ax^2+bx+c}} = \frac{1}{a} \sqrt{ax^2+bx+c} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2+bx+c}} \quad (\text{vgl. 48})$$

$$50) \int \sqrt{ax^2+bx+c} dx = \frac{2ax+b}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a} \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

D) Trigonometrische Funktionen

$$51) \int \sin ax dx = -\frac{1}{a} \cos ax$$

$$52) \int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a} \sin 2ax$$

$$53) \int \sin^3 ax dx = -\frac{1}{a} \cos ax + \frac{1}{3a} \cos^3 ax$$

$$54) \int \sin^n ax dx = -\frac{\sin^{n-1} ax \cos ax}{n a} + \frac{n-1}{n} \int \sin^{n-2} ax dx$$

$$55) \int x \sin ax dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$56) \int x^n \sin ax dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax dx$$

$$57) \int \frac{1}{\sin^2 ax} dx = \frac{1}{a} \ln \left| \tan \frac{ax}{2} \right|$$

$$58) \int \frac{1}{\sin^n ax} dx = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \int \frac{1}{\sin^{n-2} ax} dx \quad (n > 1)$$

$$59) \int \frac{1}{1 + \sin ax} dx = \frac{1}{a} \tan \left(\frac{ax}{2} - \frac{\pi}{4} \right)$$

$$60) \int \frac{1}{1 - \sin ax} dx = \frac{1}{a} \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right)$$

$$61) \int \sin ax \sin bx dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)} \quad (|a| \neq |b|)$$

$$62) \int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$63) \int \cos^2 ax \, dx = \frac{x}{2} + \frac{1}{4a} \sin 2ax$$

$$64) \int \cos^3 ax \, dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax$$

$$65) \int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{n a} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$$

$$66) \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$67) \int x^n \cos ax \, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

$$68) \int \frac{1}{\cos ax} \, dx = \frac{1}{a} \ln \left| \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right) \right|$$

$$69) \int \frac{1}{\cos^n ax} \, dx = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1} \int \frac{1}{\cos^{n-2} ax} \, dx \quad (n > 1)$$

$$70) \int \frac{1}{1 + \cos ax} \, dx = \frac{1}{a} \tan \frac{ax}{2}$$

$$71) \int \frac{1}{1 - \cos ax} \, dx = -\frac{1}{a} \cot \frac{ax}{2}$$

$$72) \int \cos ax \cos bx \, dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} \quad (|a| \neq |b|)$$

$$73) \int \sin ax \cos ax \, dx = \frac{1}{2a} \sin^2 ax$$

$$74) \int \sin ax \cos bx \, dx = -\frac{\cos (a-b)x}{2(a-b)} - \frac{\cos (a+b)x}{2(a+b)} \quad (|a| \neq |b|)$$

$$75) \int \tan ax \, dx = -\frac{1}{a} \ln |\cos ax|$$

$$76) \int \tan^2 ax \, dx = \frac{\tan ax}{a} - x$$

$$77) \int \tan^n ax \, dx = \frac{1}{a(n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax \, dx \quad (n \neq 1)$$

$$78) \int \cot ax \, dx = \frac{1}{a} \ln |\sin ax|$$

$$79) \int \cot^n ax \, dx = -\frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax \, dx \quad (n \neq 1)$$

E) Exponential- und Hyperbelfunktionen

$$80) \int e^{ax} \, dx = \frac{1}{a} e^{ax}$$

$$81) \int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$82) \int x^n e^{ax} \, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$$

$$83) \int \sinh ax \, dx = \frac{1}{a} \cosh ax$$

$$84) \int \sinh^2 ax \, dx = \frac{1}{4a} \sinh 2ax - \frac{x}{2}$$

$$85) \int \frac{1}{\sinh ax} \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right|$$

$$86) \int \cosh ax \, dx = \frac{1}{a} \sinh ax$$

$$87) \int \cosh^2 ax \, dx = \frac{1}{4a} \sinh 2ax + \frac{x}{2}$$

$$88) \int \frac{1}{\cosh ax} \, dx = \frac{2}{a} \arctan e^{ax}$$

$$89) \int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax$$

$$90) \int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax|$$

F) Gemischte Ausdrücke aus D) und E)

$$91) \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$$

$$92) \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

G) Logarithmusfunktionen

$$93) \int \ln x \, dx = x \ln x - x$$

$$94) \int (\ln x)^2 \, dx = x (\ln x)^2 - 2x \ln x + 2x$$

$$95) \int x^m \ln x \, dx = x^{m+1} \left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right) \quad (m \neq -1)$$

$$96) \int \frac{\ln x}{x^m} \, dx = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2 x^{m-1}} \quad (m \neq 1)$$

H) Arcusfunktionen

$$97) \int \arcsin \frac{x}{a} \, dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2}$$

$$98) \int \arccos \frac{x}{a} \, dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2}$$

$$99) \int \arctan \frac{x}{a} \, dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln (a^2 + x^2)$$

$$100) \int \operatorname{arccot} \frac{x}{a} \, dx = x \operatorname{arccot} \frac{x}{a} + \frac{a}{2} \ln (a^2 + x^2)$$